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The Effects of Marketable Pollution Permits on the Firm's Optimal Investment Policies

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ABSTRACT: This paper considers a firm, which has to buy marketable pollution permits in order to be allowed to pollute the environment. Pollution is an inevitable byproduct of production and in our model two ways are offered to deal with it. The first is to buy marketable permits and the second is to clean up pollution which can be achieved through investing in abatement capital stock. The problem is formulated as a dynamic model of the firm and optimal control theory is used to firm expressions for productive and abatement investments, and for equilibrium values of productive and abatement capital stock.

Finally, we determine how the firm reacts on imposition of an emissions tax and we show that an emissions tax has the same effect on the firm as marketable permits when the tax rate equals the price of a permit multiplied by the interest rate.

1. INTRODUCTION

Many production processes damage the environment and this is a subject of increasing concern in the world of today. An important question in this respect is what kind of policy instruments the government, in its role as social planner, should choose to reduce the level of pollution.

Baumol and Oates (1971) found that market-based approaches, like taxes and marketable permits, have important efficiency advantages over pollution standards, thus restricting pollution emissions directly. They derived that efficiency requires that abatement methods must be exploited such that marginal abatement costs are equal across all methods. In the case of standards it is an impossible task for the government to fix all standards such that marginal abatement costs are equal, while by imposing a tax (or creating a market for pollution permits) marginal abatement costs are automatically equalized, because all polluters will abate such that marginal abatement costs equal the tax (or the price of the permit that clears the market). This result holds under the assumption that environmental problems develop smoothly and gradually.

But in some circumstances, such as occurrence of unexpected environmental crises that require rapid changes in the rules of the control mechanism or pollution problems with threshold damage functions (e.g. Dasgupta (1982), Figure 8.3), standards can be preferable to taxes. Moreover, Buchanan and Tullock (1975) argue that firms will prefer emission standards to emission taxes because standards serve as a barrier to entry for new firms so that existing firms collect more profits. Their argument is based on the view that industry is able to exert its preference for a particular instrument because it is more likely to be well-organized than consumers. Ulph (1992) obtains that, by analyzing a multiple country game, standards should be preferred to taxes, because taxes lead to strategic interactions resulting in the choice of an inefficient level of the non-polluting input where marginal revenue is below the factor price. This will not happen when countries use standards, because producers are then precommitted to the level of the polluting input. Furthermore, marketable permits are not successful when the number of competitors is small (Hahn (1989)).

Taking all this into account we conclude that the ideal policy package contains a mixture of instruments, with taxes, marketable permits, standards, and even moral persuasion each used in certain circumstances to regulate the sources of environmental damage (cf. Baumol and Oates (1988, p. 190)). Therefore, from a management point of view it is important to know how the firm must react on imposition of each of these instruments.

This paper focuses on the effects marketable permits have on optimal dynamic firm behavior.

The implications of an emissions tax can be found in Kort, Van Loon and Luptacik (1991) and the effects of standards on the growth of the firm are analyzed in Kort (1994). Xepapadeas (1992) studies dynamic firm behavior under both a tax and an emission standard. This kind of research complements the environmental economics literature which until now is mainly concerned with market failure and public policy to correct for market externalities.

The implementation of marketable permits involves several steps (cf. Hahn (1989)). First, a target level of environmental quality is established. Next this level of environmental quality is defined in terms of total allowable emissions. Permits are then allocated to firms, with each permit enabling the owner to emit a specified amount of pollution. Firms are allowed to trade these permits among themselves.

There has been some limited experience with programs of marketable permits for the regulation of air and water quality. The major program of imposing marketable permits as a mechanism for providing economic incentives for pollution control in the USA is the Environmental Protection Agency's Emission Trading Program for the regulation of air quality (see Tietenberg (1985)).

Compared to taxes a major advantage of the marketable permit approach is that it gives the government direct control over the quantity of emissions. Under the taxes approach, the government must set a tax, and if, for example, the tax turns out to be low, pollution still exceed permissible levels. The government will find itself in the uncomfortable position of having to adjust and readjust the tax to ensure that the environment is not severely damaged (Cropper and Oates (1992)).

To study the effects of marketable permits on optimal dynamic firm behavior we first develop a dynamic model of the firm which is done in Section 2. In Section 3 the solution is obtained for a growing firm, thus a firm that starts off small. Perhaps more interesting from a practical point of view is the case where the firm is at its unregulated optimum in the beginning when regulation in the form of marketable permits goes into effect. A solution for such a firm is derived in Section 4. In Section 5 the effects of marketable permits and an emissions tax are compared, while the paper is summarized in Section 6.

2. MODEL FORMULATION

Consider a firm that has the possibility to invest in two different sorts of capital goods. One is productive but also causes pollution as an inevitable byproduct. The other one is non-productive but cleans pollution. Emissions depend linearly on both stocks of capital goods:

$$E\ (K_1, K_2) = e_1\ K_1 - e_2\ K_2 \tag{1}$$

in which:

$E\ (K_1, K_2)$: amount of emissions being a function of K_1 and K_2

$K_1 = K_1(t)$: stock of productive capital goods at time t

$K_2 = K_2(t)$: stock of abatement capital goods at time t

e_1 : emission to capital ratio of the productive capital goods ($e_1 > 0$ and constant)

e_2 : abatement to capital ratio of the abatement capital goods ($e_2 > 0$ and constant)

According to Jorgenson and Wilcoxen (1990) the most important response to environmental regulations is investment in costly new equipment for pollution abatement. Therefore in the model we give the firm the possibility to build up abatement capital, thus we introduce abatement capital stock as a state variable. Further, it is assumed that the firm starts out without having assigned any capital goods to the abatement activity yet. Both capital goods evolve according to the standard capital accumulation dynamics:

$$K_1 = I_1 - a_1\ K_1,\ K_1(0) > 0 \tag{2}$$

$$K_2 = I_2 - a_2\ K_2,\ K_2(0) = 0 \tag{3}$$

in which:

$I_1 = I_1\ (t)$: rate of investment in productive capital goods at time t

$I_2 = I_2\ (t)$: rate of investment in abatement capital goods at time t

a_1 : depreciation rate of the productive capital goods ($a_1 > 0$ and constant)

a_2 : depreciation rate of the abatement capital goods ($a_2 > 0$ and constant)

a_2 is the decay rate due to aging of pollution control capital, i.e. when filters are bought and used they wear out and per year a proportion of $100a_2\%$ has to be replaced (cf. Hartl (1992)). Stock characteristics of environmental pollution are not considered here. This is because, according to Xepapadeas (1992, p. 260), stock effects are particularly important in a model where the objective is to maximize some welfare indicator and not in a model where private profits are maximized.

Gross earnings of the firm are given by the instantaneous net revenue function $S = S\ (K_1)$. Assume that S is twice continuously differentiable, $S\ (K_1) > 0$ for $K_1 > 0$, $S'\ (K_1) > 0$, $S''\ (K_1) < 0$, $S\ (0) = 0$. [Function $S\ (K_1)$ is defmed as revenue after maximization with respect to variable inputs, e.g. labor]

Investment is costly. Let, for $i = 1,2$, $C_i\ (I_i)$ be the cost of investment with C_i a convex and increasing function, $C_i'\ (I_i) > 0$, $C_i''\ (I_i) > 0$, $C\ (0) = 0$.

To reduce pollution the government created a market where the firm must buy permits in order to be allowed to generate emissions. Pollution permits may be defmed on a temporary basis or without a time limit (cf. Siebert (1992), p. 142). We will assume here that once a permit is bought it remains valid forever.

If it has good growth prospects the firm will increase production and, after assuming for the moment that abatement capital is too costly, this will also increase emissions which implies that the firm needs to buy extra permits. These permits can be sold to other firms at the moment that the firm reduces emissions by either a sufficient increase of abatement capital stock or a reduction of production. It is assumed that one permit allows the firm to emit pollution at the rate of one unit "per period" in perpetuity. Given $K_1(0) > 0$ and $K_2(0) = 0$, $E(0) = e_1 K_1(0) > 0$ is the initial emission rate. If the firm never deviated from $E(0)$ for any $t \in [0, \infty]$ then the firm would not have to buy any more permits, so expenditures on permits would be zero. Only if the firm changes its emission rate, say positively (i.e. $\dot{E} > 0$), then the firm needs to buy more permits to support its higher rate of emissions. If the price of a permit equals p(t), then the firm's expenses on the permit market at time t equal

$$p(t)\dot{E}(t) = p(t)\left\{e_1\dot{K}_1(t) - e_2\dot{K}_2(t)\right\}. \tag{4}$$

Notice that spendings turn into receivings as soon as emissions are reduced. Whether the price of a permit will go up or down depends on the behavior of all competitors in the market. We assume that the behavior of the individual firm has no implication for the permit price. Leaving abatement activities aside for the moment, if all firms want to produce more they implicitly want to increase emissions. Therefore, the demand for emission permits goes up and the price of the permits increases. Notice in this respect that the amount of permits on the market is fixed, which in turn leads to a fixed amount of emissions generated by the whole sector.

The above description refers to a market for pollution permits that provides great flexibility due to the absence of transactions costs and other obstacles to trading. However, in practice the rules of the marketable permits can be so restrictive that the flexibility they offer is more imaginary than real (see Cropper and Oates (1992), Hahn (1989)). Nevertheless, in this paper we assume that trading barriers are absent on the permit market.

The objective of the firm is to maximize the net cash flow stream:

$$\text{maximize} \quad : \quad \int_0^\infty \left[S(K_1) - C_1(I_1) - C_2(I_2) - p\,\dot{E}(I_1, I_2, K_1, K_2)\right] \exp(-rt) \tag{5}$$

in which:

r : discount rate.

As argued by Pindyck (1991) investment expenditures are largely irreversible; that is, they are mostly sunk costs that cannot be recovered. This comes from the fact that usually capital is firm or industry specific, that is, it cannot be used productively by a different firm or in a different industry. To include irreversibility of investments in our model we add the following two non-negativity restrictions:

$$I_1 \geq 0 \tag{6}$$

$$I_2 \geq 0 \tag{7}$$

The fact that emissions cannot be negative is covered by the following state constraint:

$$e_1\, K_1 - e_2\, K_2 \geq 0 \tag{8}$$

The decision problem of the firm is to determine an investment path, $\{I_1(t), I_2(t)\}$, over an infinite planning period $[0, \infty)$, such that the objective functional in (5) is maximal, subject to the constraints (2), (3), (6), (7) and (8).

To obtain the optimality conditions we use Pontryagin's maximum principle (see e.g. Feichtinger and Hartl (1986)). The current value Hamiltonian and Lagrangian for this problem are:

$$H = S(K_1) - C_1(I_1) - C_2(I_2) - p\ \{e_2(I_2 - a_2K_2)\} + e_2(I_2 - a_2K_2) + \lambda_1(I_1 - a_lK_1) + \lambda_2(I_2 - a_2K_2) \quad (9)$$

$$L = H + \eta_1 I_1 + \eta_2 I_2 + \mu\ (e_lK_1 - e_2K_2) \quad (10)$$

in which:

λ_i : co-state variable belonging to K_i; $i = 1,2$

η_i : dynamic Lagrange multiplier belonging to the constraint $I_i \geq 0$; $i = 1,2$

μ : dynamic Lagrange multiplier belonging to the state constraint $E \geq 0$

The necessary optimality conditions are (see Feichtinger and Hartl (1986), Theorem 7.4):

$$-C_1'(I_1) - pe_1 + \lambda_1 + \eta_1 = 0 \quad (11)$$

$$-C_2(I_2) + pe_2 + \lambda_2 + \eta_2 = 0 \quad (12)$$

$$\dot{\lambda}_1 = (r + a_1)\lambda_1 - S'(K_1) - pe_1a_1 - \mu e_1 \quad (13)$$

$$\dot{\lambda}_2 = (r + a_2)\lambda_2 + pe_2a_2 + \mu e_2 \quad (14)$$

$$\eta_1 \geq 0, \eta_1 I_1 = 0 \quad (15)$$

$$\eta_2 \geq 0, \eta_2 I_2 = 0 \quad (16)$$

$$\mu \geq 0, \mu(e_1K_1 - e_2K_2) = 0 \quad (17)$$

In the direct adjoining approach that we have chosen, the co-state variables are continuous everywhere since H is strictly concave in (I_1, I_2), provided that the constraints (6), (7) and (8) are not binding at the same time (see e.g. Feichtinger and Hartl (1986), p. 168). If furthermore the transversality conditions

$$\lim_{t\to\infty} \exp(-r\,t)\lambda_i(t)\left[\widetilde{K}_i(t) - K_i(t)\right] \geq 0;\ i = 1,2 \quad (18)$$

hold for every feasible solution ($\widetilde{K}_1, \widetilde{K}_2$), then (11) - (17) are also suffcient for optimality since the maximized Hamiltonian is concave in (K_1, K_2) (Feichtinger and Hartl (1986), p. 187).

3. THE OPTIMAL INVESTMENT PATH FOR A GROWING FIRM

In this section we consider a firm that starts out with a productive capital stock that is below the steady state. First we study the case where the price of marketable permits is constant; after that we analyse what happens when permit prices increase over time, caused by an increased demand of pollution permits in a growing sector.

3.1 OPTIMAL SOLUTION IN CASE OF CONSTANT PERMIT PRICES

At the start of the planning period the firm has not invested in abatement capital stock yet, because $K_2(0) = 0$ (cf. (3)). Due to the fact that productive capital stock is positive in the beginning (cf. (2)) we conclude that emissions are positive, thus restriction (8) is not binding and $\mu = 0$ (cf. (17)). From (11) - (14) we derive that as long as $\mu = 0$ the optimal time path of (I_1, K_1) does not depend on the development of (I_2, K_2). Hence, when emission are positive we can determine the optimal investment strategies for productive and abatement capital stock independent from each other.

In this subsection we first derive the optimal productive investment strategy, after that we study the optimal abatement investment decision and in the end we put the developments of productive and abatement capital stock together. Then we can draw conclusions concerning the evolution of the amount of emissions over time and study optimal firm behaviour in case emissions become zero.

3.1.1 THE OPTIMAL PRODUCTIVE INVESTMENT DECISION

Consider investments in productive capital stock in case emissions and productive investments are positive ($\mu = \eta_1 = 0$). Differentiate (11) w.r.t time and use (13) and (11) to eliminate λ_1 and $\dot{\lambda}_1$. respectively. This yields:

$$\dot{I}_1 = \frac{1}{C_1''(I_1)}\left[(r + a_1)C_1'(I_1) + rpe_1 = S'(K_1)\right] \tag{19}$$

We wish to study the differential equation system (2) and (19) in the (K_1, I_1) phase plane. The steady state satisfies:

$$I_1^* = a_1 K_1^* \tag{20}$$

$$S'(K_1^*) = (r + a_1)C_1'(I_1^*) + rpe_1 \tag{21}$$

The economic interpretation of the steady state is as follows. The investment rate is maintaned at the constant replacement level, $a_1K_1^*$, and marginal revenue (left - hand side of (21)) equals marginal cost (right - hand side of (21)). Compared to the standard investment models wiht convex investment costs (e.g. Takayama (1985), pp. 698 - 699), here marginal cost has increased with rpe_1. This is because owning and additional unit of productive capital stock increases emissions with e_1 so that additional permints must be bought at the expense of pe_1 which in turn increases interest costs with rpe_1. Notice that buying extra permits does not leat to an increase of depreciation cost, because depreciation of productive capital stock reduces the amount of emissions.

The Jacobian determinant of the system (2), (19) and evaluated at the steady state is negative so that the steady state must be a saddle point. The phase diagram in the (K_1, I_1) - space is drawn in Figure 1. (The $\dot{I} = 0$ isocline is drawn as a straight line. This is olny for simplicity. The real shape depends on $S^{(3)}(K_1)$ and $C^{(3)}(I_1)$). Of course, this diagram only holds in case the amount of emissions is positive.

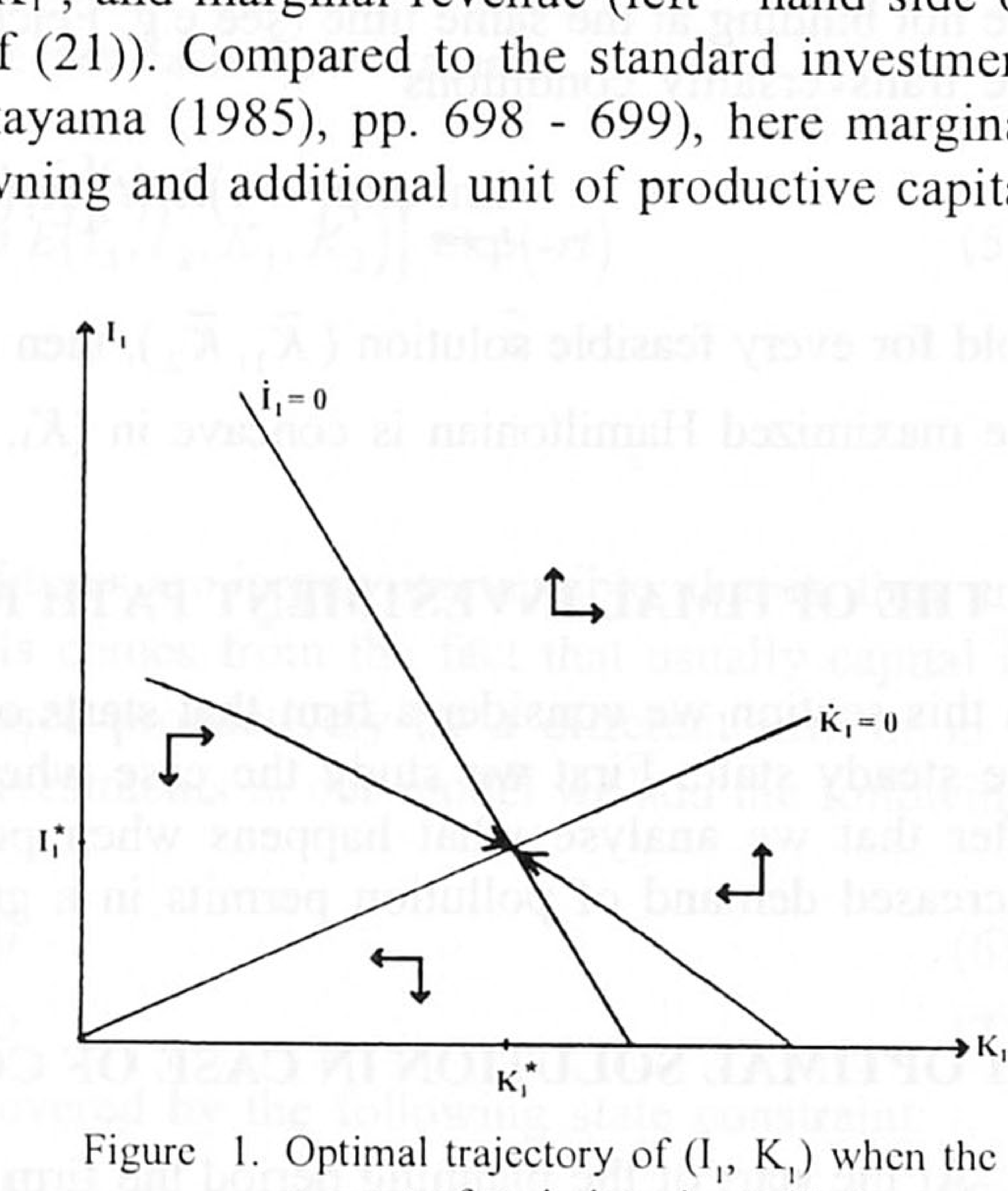

Figure 1. Optimal trajectory of (I_1, K_1) when the amount of emissions is positive.

Due to the fact that the Hamiltonian is strictly concave in (I_1,I_2) we know that investments are continous over time (e.g. Feichtinger and Hartl (1986), Corollary 6.2). Therefore, we can derive from the phase diagram that I_1 can only be zero for very high values of capital stock. These values do not occur in the optimal trajectory here, because the firm out small, i.e. $K_1(0)<K_1^*$, and then it increases capital stock until the equelibrium value K_1^* is reached.

The derive an expression for the firm's productive investment rate we now use the "net present value concept" (cf. Kort (1990)). After solving the differential equation (13) with $\mu = 0$, substituting (11) (with $\eta_1 = 0$) into this relation, and using (21) as a fixed point, we derive that at each moment of time the level of productive investment must satisfy.

$$\int_t^\infty \left\{S'(K_1(s)) + p\, e_1\, a_1\right\} \exp\left(-(a_1+r)(s-t)\right) ds - C_1'(I_1(t)) - p\, e_1 = 0 \tag{22}$$

where the left -hand side is the "net preset value of marginal investment". For an interpretation consider the acquisition of an extra unit of capital at time t. The firm incurs an extra expense at time t in amount of marginal investment cost C'_1 plus spendings on extra permits pe_1 needed to account for the additional emissions generated by this extra unit of capital. On the other hand, the marginal unit of capital generates - as of time t - a stream of cash inflows consisting of revenue from selling products (S') and revenue from seling extra marketable permits (pe_1a_1). The later arises, because extra capital at time t increases depreciation later on and this in turn decreases emissions. The cash inflow stream is corrected for depreciation by multiplication by exp $(-a_1(s-t))$ and is discounted to time t by multipilication by exp $(-r(s-t))$. Condition (22) state that the net present value of marginal investment equals zero. Hence, the optimal level of productive investment satisfies the fundamental economic principle of balancing marginal revenue with marginal expenses.

Following Nickel ((1978), p. 31) we define the "desired value" of capital stock be:

$$C_1'\left(a_1 K_1^*(t)\right) + pe_1 = \int_t^\infty \left\{S'(K_1(s)) + pe_1 a_1\right\} \exp\left(-(a_1+r)(s-t)\right) ds \tag{23}$$

From (22) and (23) we obtain that $I_1(t) = a_1K_1^*(t)$. In the phase diagram we see that I_1 decreases over time for $K_1 < K_1^*$ and converges to $a_1K_1^*$ as soon as K_1 becomes equal to K_1^*. Therefore K_1^* will decrease also until it reaches K_1^*.

Substitution of $I_1 = a_1K_1^*(t)$ into (2) gives :

$$\dot{K}_1(t) = a_1\left(K_1^*(t) - K_1(t)\right) \tag{24}$$

Hence, the firm's productive investment policy satisfies a flexible accelerator mechanism (see e.g. Gould (1968)) where the desired capital stock level decreases and converges to a constant K_1^*.

3.1.2 THE OPTIMAL ABATEMENT INVESTMENT DECISION

Consider investments in abatement capital stock in case emissions and abatement investments are positive ($\mu = \eta_2 = 0$). Differentiate (12) w.r.t. time and use (14) and (12) to eliminate $\dot{\lambda}_2$ and λ_2 respectively. This yields:

$$\dot{I}_2 = \frac{1}{C_2''(I_2)}\left[(r + a_2)C_2'(I_2) - rpe_2\right] \tag{25}$$

We study the (K_2, I_2) phase plane based on the differential equations (3) and (25). The steady state satisfies:

$$I_2^* = a_2 K_2^* \tag{26}$$

$$(r + a_2)\, C'_2\, (I_2^*) = rpe_2 \tag{27}$$

From (27) we infer that marginal cost of abatement investment equals marginal revenue. The latter consists of a decrease in interest costs which is caused by the fact that the firm needs to buy less permits, because marginal abatement investment reduces the amount of emissions.

Also here the determinant of the Jacobian of the dynamic system ((3), (25)) is negative so that the dynamics correspond to a saddle point. The phase diagram in case of positive emissions is presented in Figure 2.

From Figure 2 we infer that abatement investment is constant over time. Thus (26) holds for every time t and substitution of this equation into (3) gives:

$$\dot{K}_2\,(t) = a_2\,(K_2^* - K_2\,(t)) \tag{28}$$

Hence, the firm's optimal abatement investment policy satisfies the flexible accelerator mechanism with fixed desired level of abatement capital stock. From the phase diagram we also infer that, due to the continuity property, abatement investments never become zero when emissions remain positive.

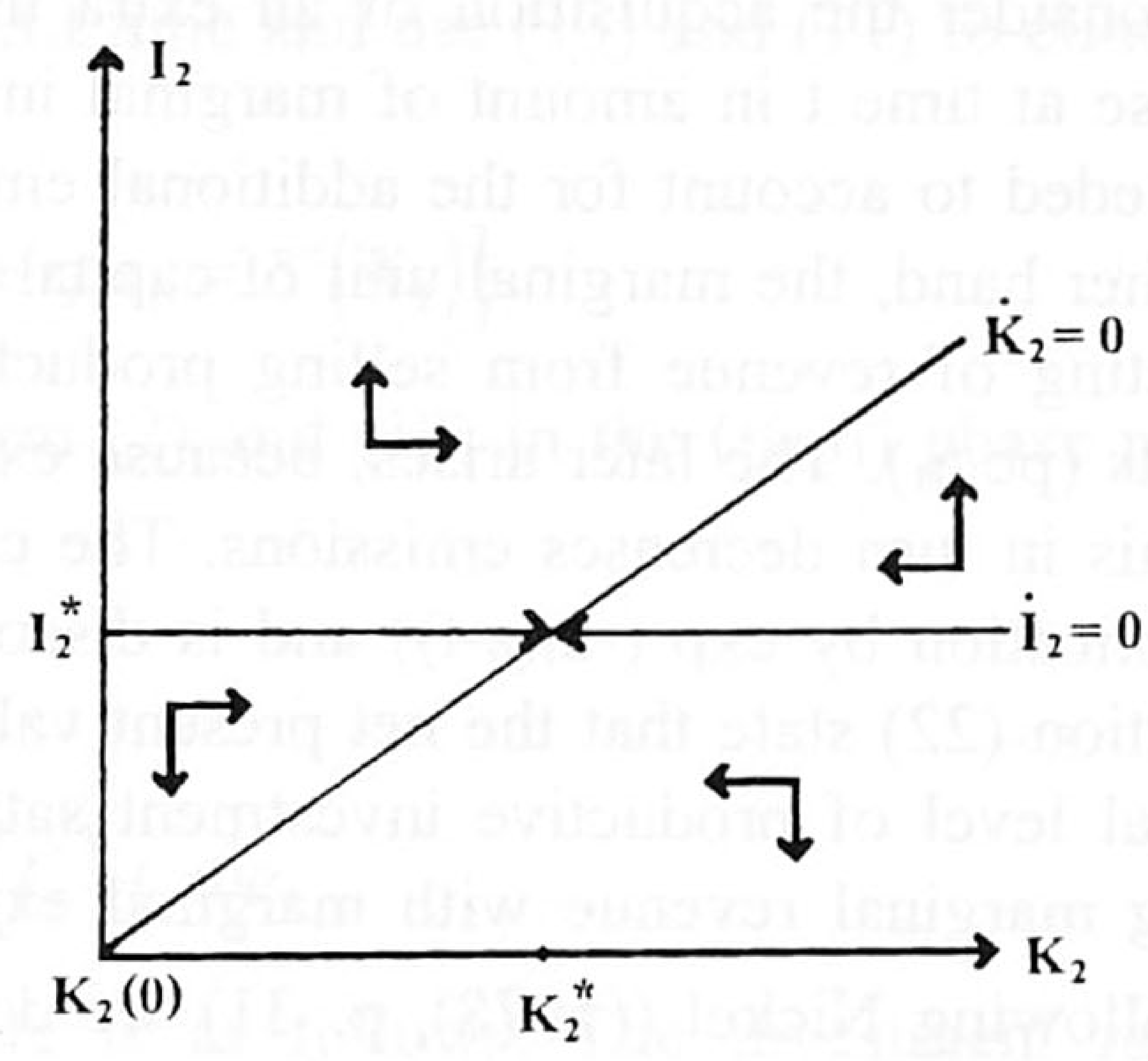

Figure 2. Optimal trajectory of (I_2, K_2) when the amount of emissions is positive.

3.1.3. DEVELOPMENT OF EMISSIONS OVER TIME

As derived before the development of both capital stocks over time satisfies:

$$\dot{K}_1(t) = a_1\left(K_1^*(t) - K_1(t)\right),\quad 0 < K_1(0) < K_1^* \tag{24}$$

$$\dot{K}_2\,(t) = a_2\,(K_2^* - K_2\,(t)),\ K_2\,(0) = 0 \tag{28}$$

in which:

$$d\,K_1^*(t)\,/\,dt < 0$$

$$\lim_{t\to\infty} K_1^*(t) = K_1^*$$

Thus productive and abatement capital stock develop according to the flexible accelerator mechanism, implying that capital stock grows faster the longer the distance from the present capital stock level towards the level of desired capital stock. If $K_1(0) < K_1^*$ the desired value $K_1^*(t)$ decreases over time and converges to the steady state value K_1^*. Then growth is even more stimulated, especially in the beginning.

From (1) we obtain that iso-pollution lines are given by:

$$K_1 = \frac{e_2}{e_1} K_2 + \frac{\bar{E}}{e_1} \tag{29}$$

in which :

$\bar{E}$: fixed amount of pollution.

Due to (29) we obtain that emissions increase over time when the firm's optimal trajectory is steeper than the iso-pollution line (Figure 3a) in the (K_2, K_1)-space. The reverse is true when the optimal trajectory is flatter than the iso-pollution line (Figure 3b). This makes sense because the optimal trajectory being flatter in the (K_2, K_1)-space implies that, given the growth of productive capital stock, abatement capital stock increases more than necessary to keep pollution on the same level. Notice that in Figure 3b the firm grows but it can nevertheless sell permits because emissions decrease over time.

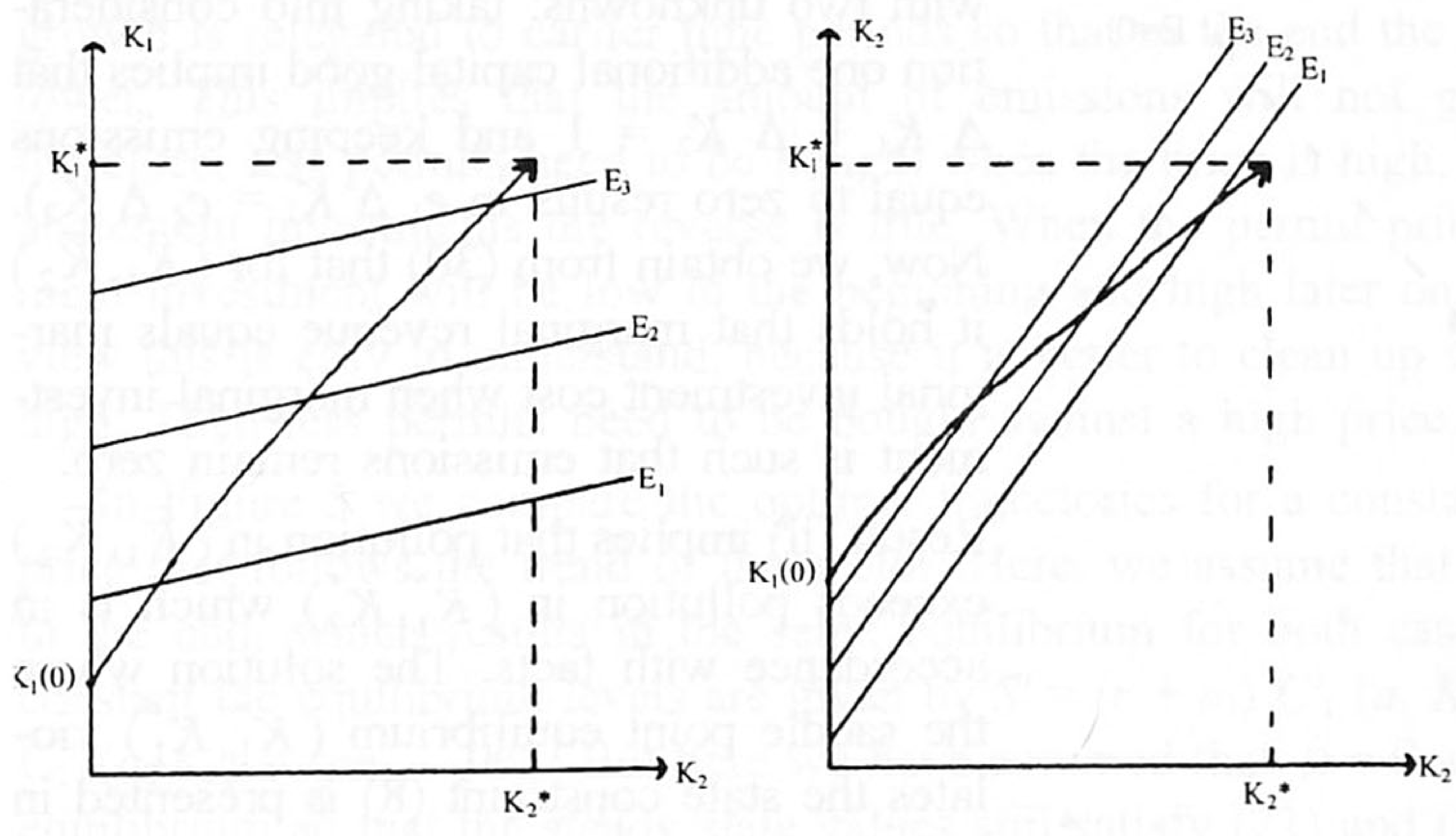

Figure 3. Optimal trajectories of (K_1, K_2) with increasing (a) and decreasing (b), where for the iso-pollution lines it holds that $E_1 < E_2 < E_3$.

In Figures 3a and 3b environmental pollution behaves monotonically. However, it is also possible that the trajectory intersects the same iso-pollution line more than once. As explained above this depends on how the slope of the trajectory develops in the (K_2, K_1)-space.

In Figure 3b it can happen that emissions decrease that fast that they become zero before the firm has reached its equilibrium. This implies that emissions are negative in (K_1^*, K_2^*) so that the state constraint (8) is violated in this point. Therefore the optimal trajectory must be different in this case. To obtain the new optimal trajectory we state the following proposition.

Proposition 1

In case emissions are negative in the saddle point equilibrium (K_1^*, K_2^*) the firm will ultimately reach another steady state $\left(\hat{K}_1, \hat{K}_2\right)$ which is situated on the iso-pollution line where emissions are zero. This new steady state and the optimal trajectory towards this point have the following properties:

i) $\hat{K}_1$ and $\hat{K}_2$ satisfy:

$$\frac{e_2}{e_1+e_2} S'(\hat{K}_1) = \frac{e_2}{e_1+e_2}(r+a_1)C_1'(a_1\hat{K}_1) + \frac{e_1}{e_1+e_2}(r+a_2)C_2'(a_2\hat{K}_2) \quad (30)$$

ii) $\hat{K}_1 > K_1^*, \hat{K}_2 < K_2^*$

iii) The optimal trajectory passes into the $E = 0$ - line such that $d K_1/d K_2$ is continuous. It cannot happen that the trajectory intersects the $E = 0$ - line exactly at $(\hat{K}_1, \hat{K}_2)$.

Proof

See Appendix.

For an interpretation of equation (30) consider an increase of capital goods with one unit when the firm is at the steady state $(\hat{K}_1, \hat{K}_2)$. To keep emissions equal to zero $e_2/(e_1 + e_2)$ is added to K_1 and $e_1/(e_1 + e_2)$ to K_2 (this division is the result of solving two equations with two unknowns: taking into consideration one additional capital good implies that $\Delta K_1 + \Delta K_2 = 1$ and keeping emissions equal to zero results in $e_1 \Delta K_1 = e_2 \Delta K_2$). Now, we obtain from (30) that for $(\hat{K}_1, \hat{K}_2)$ it holds that marginal revenue equals marginal investment cost when marginal investment is such that emissions remain zero.

Result (ii) implies that pollution in $(\hat{K}_1, \hat{K}_2)$ exceeds pollution in (K_1^*, K_2^*) which is in accordance with facts. The solution where the saddle point equilibrium (K_1^*, K_1^*) violates the state constraint (8) is presented in Figure 4.

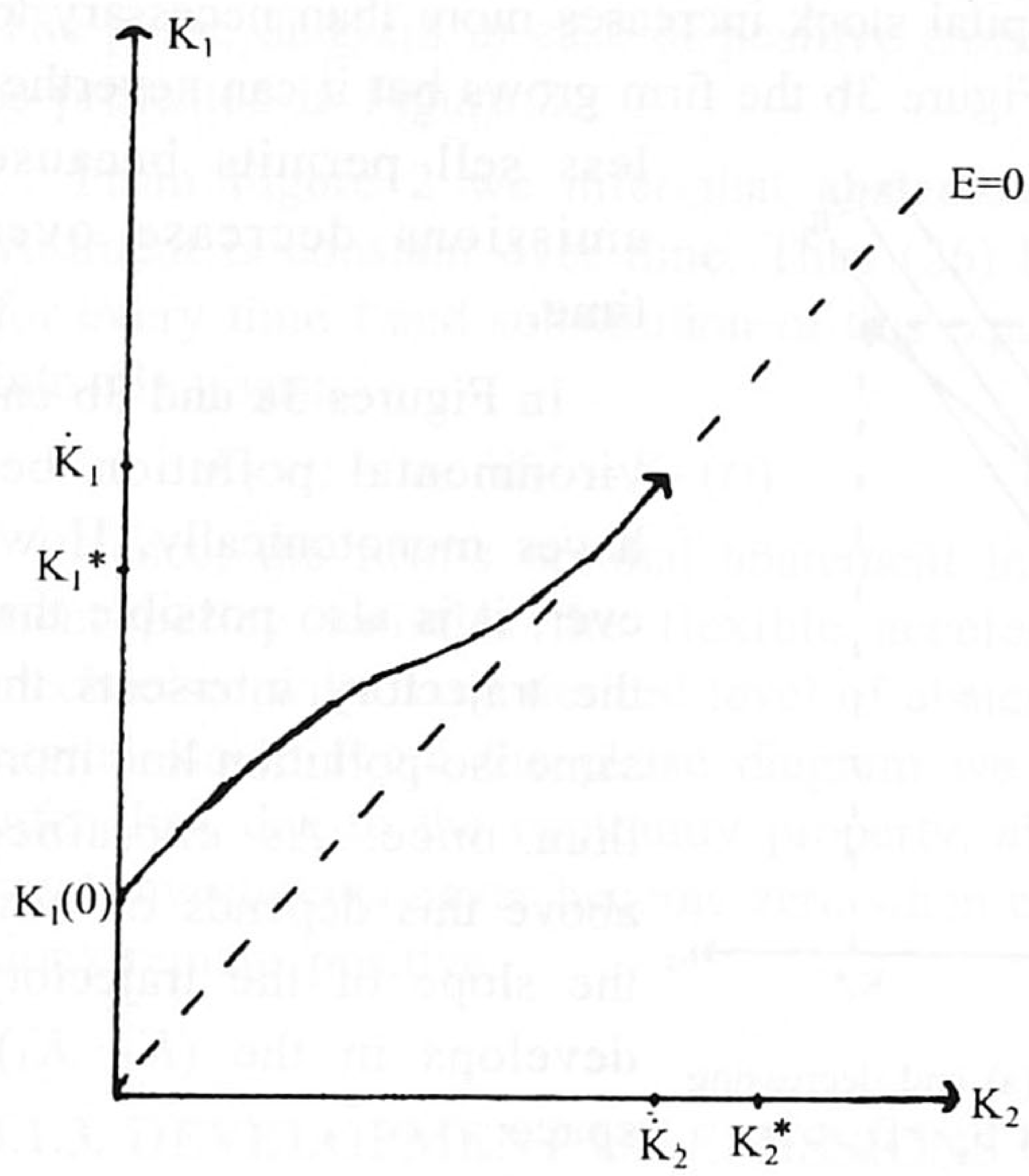

Figure 4. Optimal trajectory when (K_1^*, K_2^*) violates the state constraints (8).

3.2 OPTIMAL SOLUTION WHEN PERMIT PRICES ARE NOT CONSTANT.

The influence of varying prices of pollution permits on the firm's investment policy can best be inferred by using the concept net present value of marginal investment. This is because the canonical system is not autonomous anymore and therefore the phase plane results cannot be used. Without loss of generality we restrict ourselves to the case where emissions and investments are positive, thus $\mu = \eta_1 = \eta_2 = 0$. Then for productive investments the net present value is represented by (22) in case of constant prices. When permit prices vary this formula becomes:

$$\int_t^\infty \left\{S'(K_1(s)) + p(s)\, e_1 a_1\right\} \exp\left(-(a_1+r)(s-t)\right)ds - C_1'(I_1(t)) - p(t)e_1 = 0 \quad (31)$$

From (12) (with $\eta_2 = 0$), (14) and (27) we obtain the following net present value equation for marginal abatement investment:

$$\int_t^\infty -p(s)\, e_2 a_2 \, \exp\left(-(a_2+r)(s-t)\right) ds - C_2'(I_2(t)) + p(t)\, e_2 = 0 \qquad (32)$$

In order to obtain some results on how changes in permit prices influence the behavior of the firm let us suppose that this firm is representative for the whole sector. Consider the solution of Figure 3a where the firm grows and also emissions increase over time until the equilibrium is reached. Then capital stock as well as emissions remain constant. Because the firm is assumed to be representative it also holds for all firms in the sector that the demand for pollution permits is low in the beginning but goes up later on. Therefore, the price of the permits that clears the market will be low in the beginning, it will increase over time, and it will end up at a constant level as soon as the firms have reached their stationary equilibrium.

Interpreting (31) we may conclude that higher permit prices in the future imply that productive investments increase in the beginning. Economically this can be explained by noting that it is less costly to increase pollution when the permit price is still low. Thus growth is relegated to earlier time periods so that in the end the productive investment rate is lower. This implies that the amount of emissions will not grow so much later on and, therefore, less permits need to be bought when the price is high. From (32) we obtain that for abatement investments the reverse is true. When the permit price increases over time abatement investment will be low in the beginning and high later on. From an economic point of view this is easy to understand, because it is better to clean up when the price of pollution is high. Then less permits need to be bought against a high price.

In Figure 5 we compare the optimal trajectories for a constant permit price and a permit price that follows the trend of the sector. Here, we assume that the permit price is the same in the end, which results in the same equilibrium for both cases. (Actually, when p is not constant the equilibrium levels are given by $S' = (r + a_1)\, C'_1\, (a_1 K_1^*) + rpe_1 - \dot{p}e_1$ and $(r + a_2)\, C_2'\, (a_2 K_2) = rpe_2 - \dot{p}e_2$. But here we have assumed that $\dot{p} = 0$ as soon as the firm reaches its equilibrium so that the steady state values still satisfy (21) and (27)). We conclude that under a first increasing and then constant permit price pollution increases more in the beginning but arrives at the same level in the end. Notice that it is even possible that pollution decreases when the equilibrium level is approached, which will be the case when the optimal trajectory is flatter than the iso-pollution line. Hence, buying pollution permits is relegated to the beginning of the planning period, i.e. when the price is low.

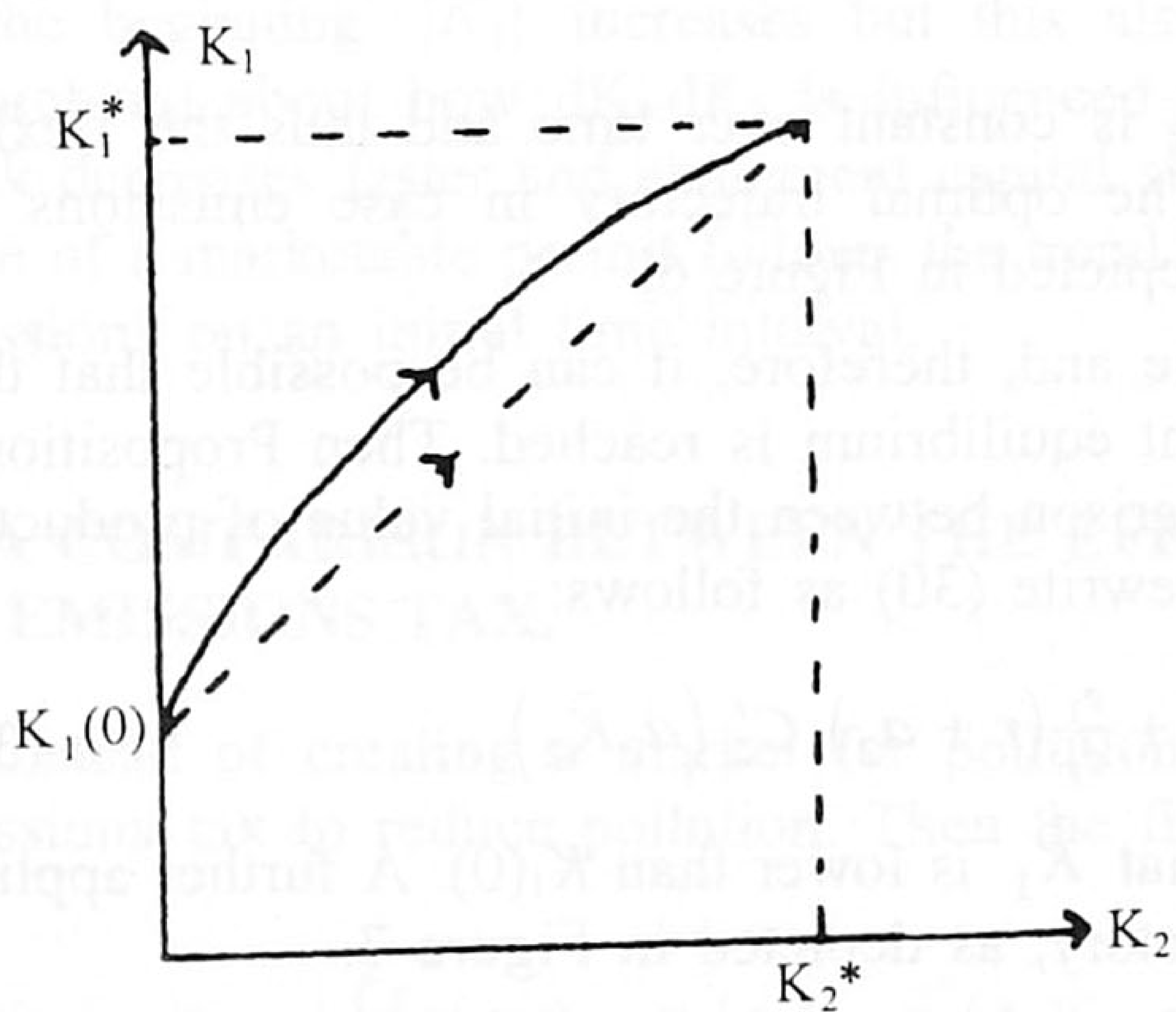

Figure 5. Optimal trajectory where the permit price is constant (--) or follows the trend of the sector (-).

Notice that the effect shown in Figure 5 will be intensified when all firms in the sector act the same way. Then we get a faster increase of total pollution in the beginning, resulting in a faster increase of the price of pollution permits. Then, according to (31) and (32), productive capital stock increases more and abatement investments are lower in the beginning.

4. THE OPTIMAL INVESTMENT PATH FOR A CONTRACTING FIRM.

Here we consider the practically interesting case where the firm finds itself at the unregulated steady state capital stock level when pollution regulation goes into effect (this is unlike Hartl (1992) who analyzes the case where the firm anticipates beforehand on the moment that regulation comes into force). Then according to the standard investment literature (e.g. Nickell (1978)) the initial level of productive capital stock satisfies:

$$S'(K_1(0)) = (r + a_1)\, C_1'(a_1 K_1(0)) \tag{33}$$

Also here we still assume that at time point zero the firm has not invested in abatement capital stock yet, thus $K_2(0) = 0$.

Like in the previous section, let us first suppose that the price of a marketable permit is constant over time. Then, when emissions are positive over the whole planning period, Figures 1 and 2 are still valid.

From (21) and (33) we infer that the equilibrium level is below the initial level of productive capital stock. Due to Figure 1 we know that then productive investments increase over time and that it is even possible that there is an initial time period where productive investments are zero. From (11), (13) (with $\mu = 0$) and (21) we obtain that this is the case when it holds that:

$$C_1'(0) + p\,e_1 > \int_t^{\infty} \left\{ S'(K_1(s)) + p\,e_1 a_1 \right\} \exp\left(-(a_1 + r)(s - t)\right) ds \tag{34}$$

Hence, marginal expenses exceed future cash intlow due to marginal productive investment. Then the net present value of marginal investment is negative, implying that it is optimal to put productive investments at their lower bound, i.e. investments are zero (cf. Kort (1990)).

When I_1 is not zero the optimal productive investment policy satisfies the flexible accelerator mechanism (24), but the difference is that now $K_1^*(t)$ increases over time, due to the fact that I_1 increases (cf. (2), (24)).

According to Figure 2 we conclude that I_2 is constant over time and thus the flexible accelerator mechanism (28) still holds too. The optimal trajectory in case emissions are positive during the whole planning period is depicted in Figure 6.

It is clear that emissions decrease over time and, therefore, it can be possible that they decrease to zero already before the saddle point equilibrium is reached. Then Proposition 1 becomes valid again. In order to make a comparison between the initial value of productive capital stock and the new steady state $\hat{K}_1$ we rewrite (30) as follows:

$$S'(\hat{K}_1) = (r + a_1)\, C_1'(a_1 \hat{K}_1) + \frac{e_1}{e_2}(r + a_2)\, C_2'(a_2 \hat{K}_2) \tag{35}$$

Now we can conclude from (33) and (35) that $\hat{K}_1$ is lower than $K_1(0)$. A further application of Proposition 1 leads to the optimal trajectory, as depicted in Figure 7.

At the end of this section we study how the firm's investment policy is influenced when the permit price follows the trend of the sector in stead of being constant. Like in Subsection 3.2 we assume that the development of the firm is symptomatic for the whole sector, implying that the

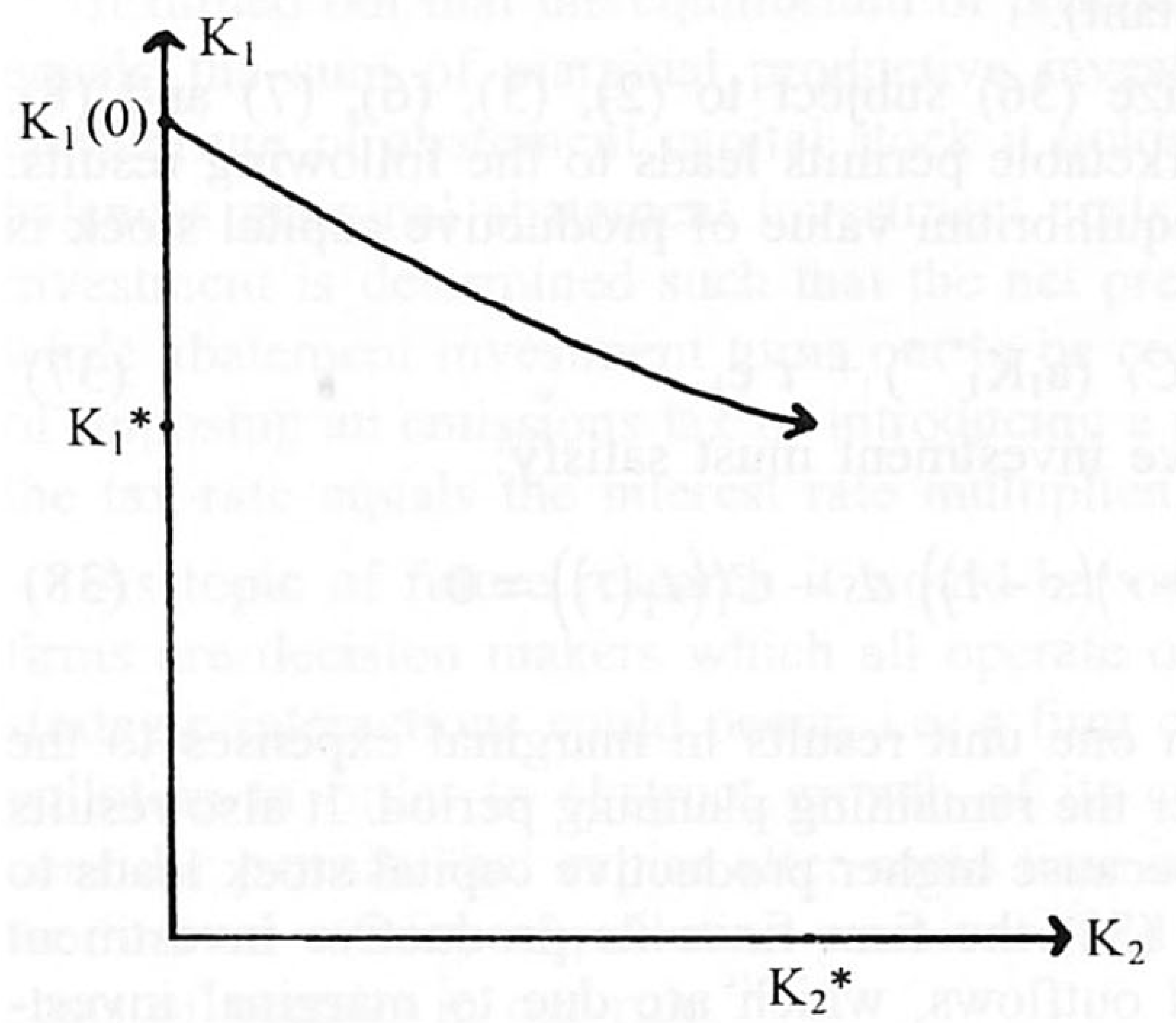

Figure 6. Optimal trajectory in case the firm starts out at its unregulated optimum.

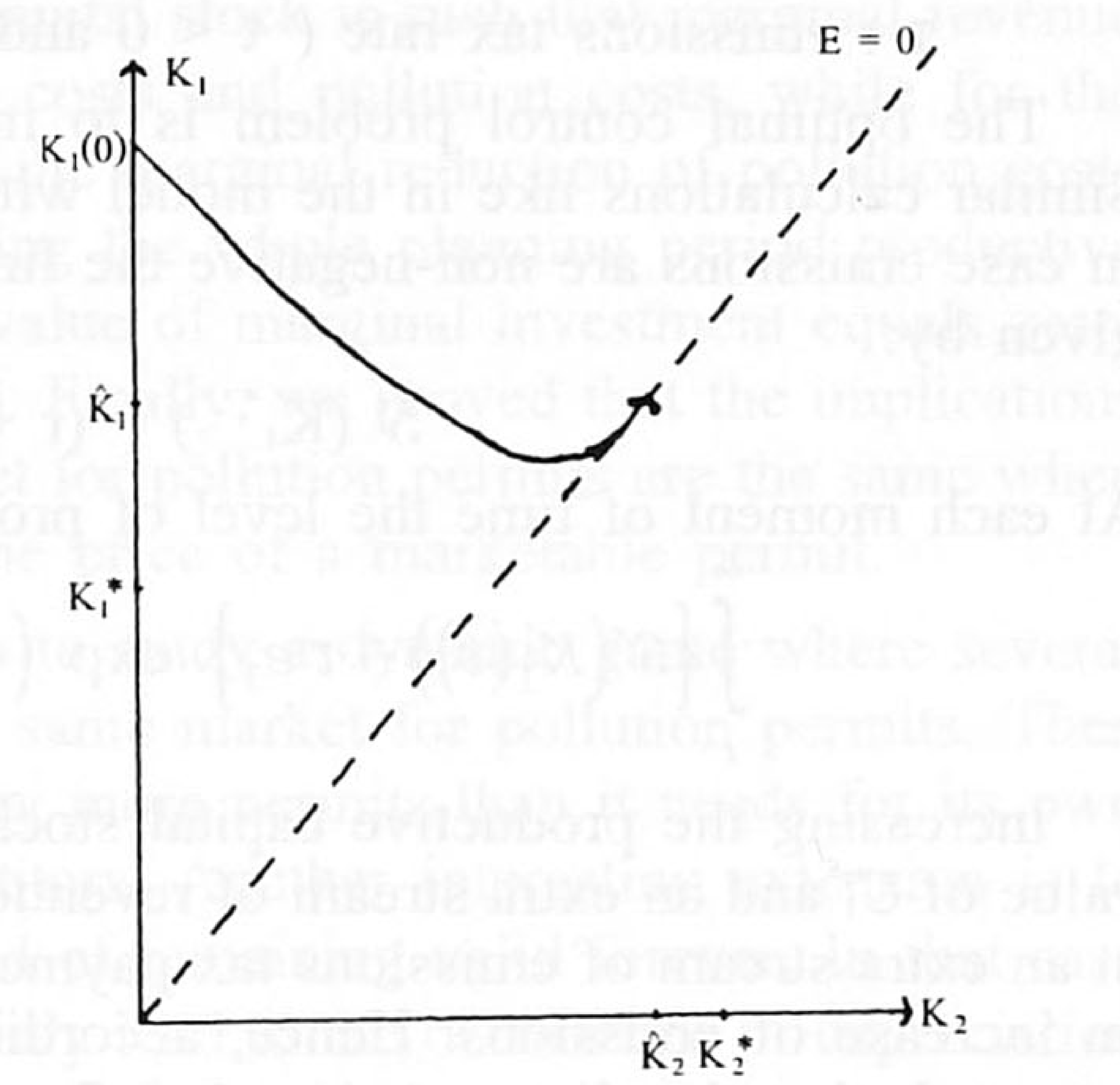

Figure 7. Optimal trajectory in case the firm starts out at its unregulated optimum and when (K_1^*, K_2^*) violates the state constraints (8).

demand for permits decreases until this demand becomes constant when the firms have reached their equilibrium value. We assume here that all firms behave like in Figure 6, leaving the possibility of emissions becoming equal to zero aside. The price of a permit will diminish over time as long as the demand for pollution permits decreases.

As can be inferred from (31) a lower permit price in future implies a decrease of productive investments in the beginning. An economic reason for this behavior is that as long as the permit price is high it is better for the firm to increase the pollution reduction, and this leads to the decrease of productive investments.

Of course, abatement investments are more profitable when the cost of pollution is high, thus when the price of marketable permits is high. Therefore, compared to the case of constant permit price, the firm will increase abatement investment in the beginning of the planning period and reduce them later on.

How does the optimal trajectory change compared to the case of a constant permit price? In the beginning $|K_1|$ increases but this also holds for K_2 so that nothing can be said beforehand about how dK_1/dK_2 is influenced. But we *can* conclude that productive capital stock decreases faster and abatement capital stock increases faster in the beginning when the price of a marketable permit follows the trend of the sector. This leads to a faster decrease of emissions on an initial time interval.

5. A COMPARISON BETWEEN THE EFFECTS OF MARKETABLE PERMITS AND AN EMISSIONS TAX.

Instead of creating a market for pollution permits the government can also impose an emissions tax to reduce pollution. Then the firm's objective function becomes:

$$\text{maximize}: \int_t^{\infty} \left[S(K_1) - C_1(I_1) - C_2(I_2) - \tau(e_1K_1 - e_2K_2)\right] \exp(-rt)\, dt \tag{36}$$

in which :

τ : emissions tax rate ($\tau < 0$ and constant).

The optimal control problem is to maximize (36) subject to (2), (3), (6), (7) and (8). Similar calculations like in the model with marketable permits leads to the following results. In case emissions are non-negative the firm's equilibrium value of productive capital stock is given by:

$$S'(K_1^{**}) = (r + a_1)\, C_1'(a_1 K_1^{**}) + \tau e_1 \tag{37}$$

At each moment of time the level of productive investment must satisfy:

$$\int_t^\infty \{S'(K_1(s)) - \tau e_1\} \exp\left(-(a_1 + r)(s - t)\right) ds - C_1'(I_1(t)) = 0 \tag{38}$$

Increasing the productive capital stock with one unit results in marginal expenses to the value of C_1' and an extra stream of revenues over the remaining planning period. It also results in an extra stream of emissions tax payments, because higher productive capital stock leads to an increase of emissions. Hence, according to (38) the firm fixes its productive investment rate such that the discounted cash inflows and outflows, which are due to marginal investment, are balanced. Abatement investments are constant and determined by the following equation:

$$(r + a_2)\, C_2'(I_2) = \tau e_2 \tag{39}$$

In case emissions are negative in the saddle point equilibrium, the new steady state is also here represented by (30).

After comparing (21), (22) and (27) with (37) - (39) we can conclude that marketable permits and an emissions tax lead to the same results if it holds that:

$$rp = \tau \tag{40}$$

Here it is important to notice that (22) can be rewritten into:

$$\int_t^\infty S'(K_1(s)) \exp\left(-(a_1 + r)(s - t)\right) ds - C_1'(I_1(t)) - \frac{rpe_1}{a_1 + r} = 0$$

And (38) is equivalent with:

$$\int_t^\infty S'(K_1(s)) \exp\left(-(a_1 + r)(s - t)\right) ds - C_1'(I_1(t)) - \frac{\tau e_1}{a_1 + r} = 0$$

Alternatively, this can be proved by comparing the system of differential equations for (K_1, I_1) and (K_2, I_2). To understand equation (40) consider the marginal pollution costs under both instruments. If during a certain period with one unit time length the firm owns one extra unit of productive capital stock, emissions are increased with e_1. In case the government has imposed an emissions tax the firm has to pay an extra tax to the value of τe_1.

Under marketable permits the firm has to buy extra permits at the expense of pe_1 in order to be allowed to increase emissions with el during that period. After the period is over these permits can be sold again. Hence, the firm needs pe_1 units of money only during this time period which results in interest costs that equal rpe_1.[1)]

6. SUMMARY

In this paper we studied the effects marketable permits have on the firm's dynamic investment policy. It was assumed that the firm can invest in capital stock that is used to produce goods and it can also invest in a second kind of capital stock through which the firm can reduce its emissions.

It turned out that the equilibrium of productive capital stock is such that marginal revenue equals the sum of marginal productive investment costs and pollution costs, while for the equilibrium of abatement capital stock it holds that the marginal reduction of pollution costs balances marginal abatement investment costs. During the whole planning period productive investment is determined such that the net present value of marginal investment equals zero, while abatement investment turns out to be constant. Finally, we proved that the implications of imposing an emissions tax or introducing a market for pollution permits are the same when the tax rate equals the interest rate multiplied by the price of a marketable permit.

As topic of future research it would be valuable to study a dynamic game where several firms are decision makers which all operate on the same market for pollution permits. Then strategic interactions could occur, i.e. a firm can buy more permits than it needs for its own pollution in order to obstruct growth of its competitors. Another interesting extension is to consider permits that expire after some time in stead of remaining valid forever. In that case the limited validity of pollution permits would certainly lead to the occurrence of depreciation costs of permits in our rule.[2)]

APPENDIX. Proof of Proposition I

To calculate the steady state when $E = 0$ we put $\eta_1 = \eta_2 = \dot{\lambda}_1 = \dot{\lambda}_2 = 0$, substitute (1) in (3), (2) in (4), and obtain:

$$e_1\mu = (r + a_1)C_1'(a_1\hat{K}_1) + rpe_1 - S'(\hat{K}_1) \tag{A1}$$

$$e_2\mu = rpe_2 - (r + a_2)\, C_2'(a_2\hat{K}_2) \tag{A2}$$

Notice that in case of $E = 0$ the state constraint (8) is binding so that m can be positive. If we multiply (A2) with $e_1/(e_1 + e_2)$ and subtract this from (A1) $xe_2/(e_1 + e_2)$, we obtain property (i) of Proposition 1.

To determine the position of $(\hat{K}_1, \hat{K}_2)$ relative to the saddle point equilibrium (K_1^*, K_2^*), first observe that from (21) and (27) we can derive:

$$\tfrac{e_2}{e_1+e_2} S'(K_1^*) - \tfrac{e_2}{e_1+e_2} C_1'(a_1K_1^*) - \tfrac{e_1}{e_1+e_2} C_2'(a_2K_2^*) = 0 \tag{A3}$$

From (30) and (A3) we conclude that both $(\hat{K}_1, \hat{K}_2)$ and (K_1^*, K_2^*) are situated on the following curve in the (K_2, K_1)-space:

$$\tfrac{e_2}{e_1+e_2} S'(K_1) - \tfrac{e_2}{e_1+e_2} C_1'(a_1K_1) - \tfrac{e_1}{e_1+e_2} C_2'(a_2K_2) = 0$$

[1)] In case permits are only valid during a finite time-interval pollution costs not only consist of interest costs, but also of depreciation costs in order to account for the fact that the value of a marketable permit decreases over time.

[2)] The author thanks Antoon van den Elzen, Richard Hartl, Piet Verheyen and two anonymous referees for providing valuable comments. This research has been made possible by a fellowship of the Royal Netherlands Academy of Arts and Sciences.

This curve has a negative slope and, after taking into account that emissions are zero in $(\hat{K}_1, \hat{K}_2)$ and negative in (K_1^*, K_2^*), we conclude that property (ii) of Proposition 1 must be valid.

Due to (11) and (12) we obtain that continuity of l_1 and l_2 implies that I_1 and I_2 are also continuous. This in turn implies that $\dot{K}_1$ and $\dot{K}_2$ are continuous, which means that dK_1/dK_2 is continuous or that $\dot{K}_1$ and $\dot{K}_2$ both converge to zero when the trajectory intersects the $E = 0$ - line. In the latter case this would mean that the intersection takes place at $(\hat{K}_1, \hat{K}_2)$.

Now let us assume that the trajectory intersects the $E = 0$ - line at $(\hat{K}_1, \hat{K}_2)$, and that the intersection happens, say, at time point $\hat{t}$. For every $t \geq \hat{t}$ $(A1)$ *and* $(A2)$ hold, and we can use (20), (21), (27) and property (ii) of Proposition 1 to verify that m > 0. We now obtain from (13) that $\dot{\lambda}_1$ jumps downwards at $t = \hat{t}$ and, after differentiation of (11) w.r.t. time, we can conclude that $\dot{I}_1$ jumps downwards too. Because the steady state is reached at $t = \hat{t}$, $\dot{I}_1$ equals zero for $t \geq \hat{t}$ and thus the downward jump at $\hat{t}$ implies that $\dot{I}_1 > 0$ at $\hat{t} - \in$ ($\in$ small).

But, because K and I are continuous over time, we can obtain from (19), (20), (21) and property (ii) of Proposition 1 that $\dot{I}_1 < 0$ at $\hat{t} - \in$.

Due to this contradiction we can conclude that the optimal trajeetory cannot intersect the E = 0 - line exactly at $(\hat{K}_1, \hat{K}_2)$.

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